Math 2130
Hw 8 Solutions

1

$$x = t^2 \longrightarrow x' = 2t$$

$$y = t \longrightarrow y' = 1$$

$$ds = \sqrt{(x')^2 + (y')^2}\, dt = \sqrt{(2t)^2 + (1)^2}\, dt$$

$$\int_C y\, ds = \int_0^2 \underbrace{t}_{y}\, \underbrace{\sqrt{4t^2+1}\, dt}_{ds}$$

$$u = 4t^2 + 1$$
$$du = 8t\,dt$$
$$\tfrac{1}{8} du = t\,dt$$
$$t = 0 \to u = 1$$
$$t = 2 \to u = 4 \cdot 2^2 + 1 = 17$$

$$= \int_1^{17} \frac{1}{8}\sqrt{u}\, du = \frac{1}{8}\int_1^{17} u^{1/2}\, du$$

$$= \frac{1}{8}\, \frac{u^{3/2}}{3/2}\Bigg|_{u=1}^{17}$$

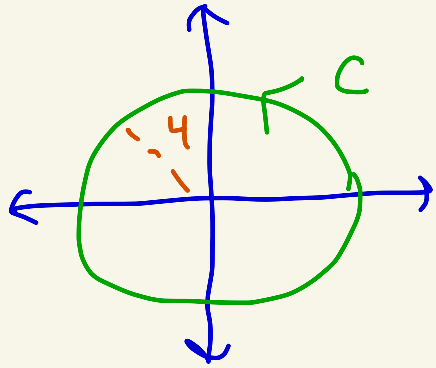
$$= \frac{1}{8} \cdot \frac{2}{3}\left[17^{3/2} - 1^{3/2}\right]$$

$$= \boxed{\frac{1}{12}\left[17^{3/2} - 1\right]}$$

②

C is parameterized by

$x = 4\cos(t)$
$y = 4\sin(t)$
$0 \le t \le 2\pi$



$x' = -4\sin(t)$
$y' = 4\cos(t)$

$$ds = \sqrt{(x')^2 + (y')^2}\,dt = \sqrt{(-4\cos(t))^2 + (4\sin(t))^2}\,dt$$

$$= \sqrt{4^2 \underbrace{(\cos^2(t) + \sin^2(t))}_{1}}\,dt = 4\,dt$$

$$\int_C (x^2 + y^2)\,ds = \int_0^{2\pi} \Big(\underbrace{(4\cos(t))^2 + (4\sin(t))^2}_{x^2+y^2}\Big) \cdot 4\,dt$$

$$= \int_0^{2\pi} \Big[4^2 \underbrace{(\cos^2(t) + \sin^2(t))}_{1}\Big] \cdot 4\,dt$$

$$= 4^3 \int_0^{2\pi} 1\,dt = 64\,t\Big|_0^{2\pi}$$

$$= 64(2\pi - 0)$$

$$= \boxed{128\pi}$$

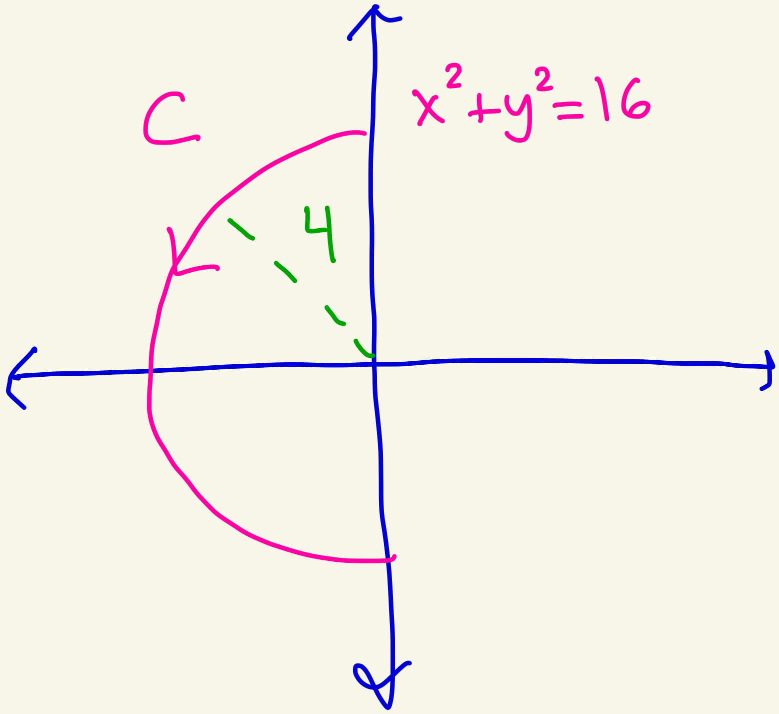
3

C is parameterized by

$x = 4\cos(t)$

$y = 4\sin(t)$

$\pi \le t \le 2\pi$



$x' = -4\sin(t)$

$y' = 4\cos(t)$

$$ds = \sqrt{(x')^2 + (y')^2}\; dt$$

$$= \sqrt{(-4\sin(t))^2 + (4\cos(t))^2}\; dt$$

$$= \sqrt{4^2(\underbrace{\sin^2(t) + \cos^2(t)}_{1})}\; dt = 4\,dt$$

$$\int_C x y^4\, ds = \int_{\pi}^{2\pi} \underbrace{4\cos(t)\left[4\sin(t)\right]^4}_{xy^4} \cdot \underbrace{4\,dt}_{ds}$$

$$= 4^6 \int_{\pi}^{2\pi} \sin^4(t)\cos(t)\,dt$$

$$= 4096 \int_0^0 u^4\, dt = 4096 \cdot \frac{u^5}{5}\Big|_{u=0}^{0} = \frac{4096}{5}(0-0)$$

$$= \boxed{0}$$

$u = \sin(t)$, $du = \cos(t)\,dt$ | $t=\pi \to u = \sin(\pi) = 0$, $t = 2\pi \to u = \sin(2\pi) = 0$

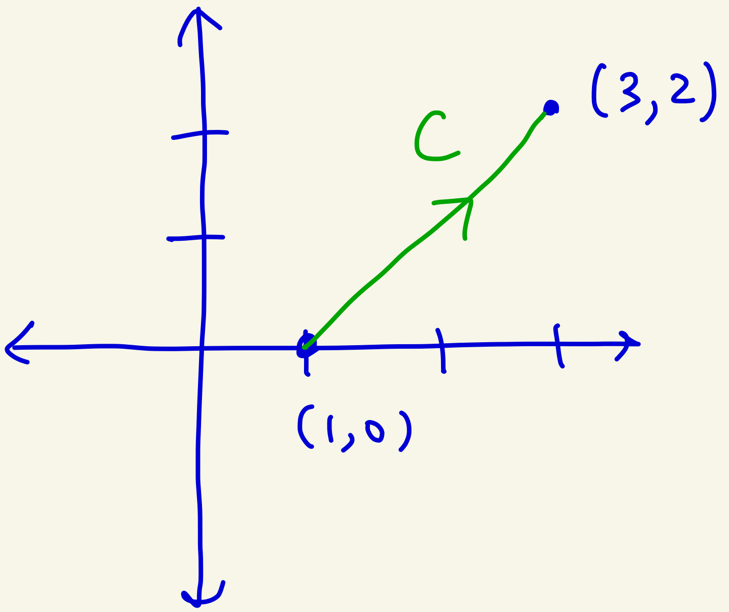
4

C is parameterized by

$x = 1 + (3-1)t = 1 + 2t$

$y = 0 + (2-0)t = 2t$

$0 \le t \le 1$



Then,

$x' = 2$

$y' = 2$

$ds = \sqrt{(x')^2 + (y')^2}\, dt = \sqrt{2^2 + 2^2}\, dt = \sqrt{8}\, dt$

$$\int_C (2x-y)\, ds = \int_0^1 \underbrace{\left(2(1+2t) - (2t)\right)}_{2x-y} \underbrace{\sqrt{8}\, dt}_{ds}$$

$$= \sqrt{8} \int_0^1 (2t+2)\, dt = \sqrt{8} \left[t^2 + 2t \Big|_0^1 \right]$$

$$= \sqrt{8} \left[(1^2 + 2) - (0^2 + 0) \right] = \boxed{3\sqrt{8}}$$

5

C is the line segment from $(0,0,0)$ to $(1,2,3)$.
It is parameterized by

$$x = 0 + (1-0)t = t$$
$$y = 0 + (2-0)t = 2t$$
$$z = 0 + (3-0)t = 3t$$
$$0 \leq t \leq 1$$

Then:

$$x' = 1, \quad y' = 2, \quad z' = 3$$
$$ds = \sqrt{(x')^2 + (y')^2 + (z')^2}\, dt = \sqrt{1^2 + 2^2 + 3^2}\, dt = \sqrt{14}\, dt$$

So,

$$\int_C x e^{yz}\, ds = \int_0^1 \underbrace{t e^{(2t)(3t)}}_{xe^{yz}} \underbrace{\sqrt{14}\, dt}_{ds} = \sqrt{14} \int_0^1 t e^{6t^2}\, dt$$

$$= \sqrt{14} \int_0^6 \frac{1}{12} e^u\, du = \frac{\sqrt{14}}{12} e^u \Big|_{u=0}^{6}$$

$$= \frac{\sqrt{14}}{12}\left(e^6 - e^0\right)$$

$$= \boxed{\frac{\sqrt{14}}{12}\left(e^6 - 1\right)}$$

$u = 6t^2$
$du = 12t\,dt$
$\frac{1}{12}du = t\,dt$
$t = 0 \rightarrow u = 6 \cdot 0^2 = 0$
$t = 1 \rightarrow u = 6 \cdot 1^2 = 6$